ON THE DYNAMICAL PROPERTIES OF GEOMAGNETIC INDICES FOR SPACE WEATHER PURPOSES

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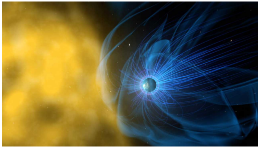
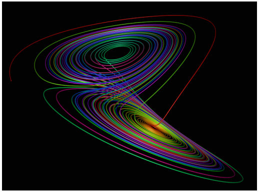
1. THE DYNAMICAL SYSTEM CALLED MAGNETOSPHERE



2. THE MULTISCALE NATURE OF THE MAGNETOSPHERE

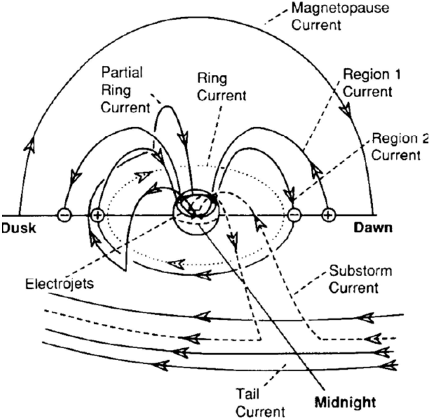
3. THE CHAOTIC/COMPLEX MAGNETOSPHERE



4. TIPS & CONCLUSIONS

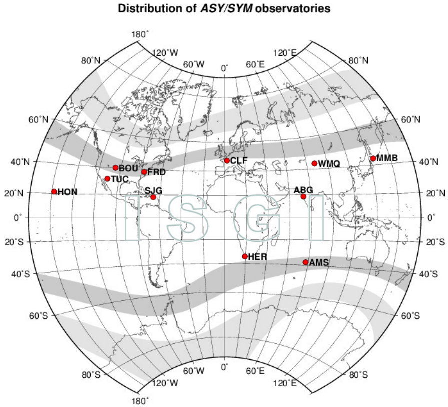
THE MAGNETOSPHERE ...

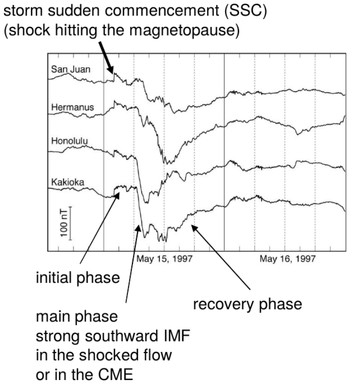
- is a highly dynamic complex system (Tsurutani et al., 1990; Vassiliadis et al., 1990)
- manifests multiscale dynamics with scale-invariant features (Consolini, 2002; Uritsky et al., 2002)
- is in a far-from-equilibrium near-critical state (Chang et al., 1992)



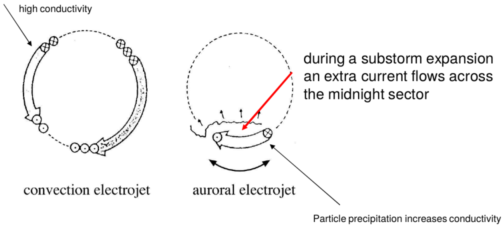
The inherent multiscale and near criticality character of the magnetospheric dynamics can give rise to some critical issues in the right forecast of the geomagnetic response to solar wind changes, especially at the short timescales, that is, at timescales of the order of few minutes that are strongly affected by the above phenomena.

- The ring current activity is monitored by the low-latitude geomagnetic index known as SYM-H
- The SYM-H index (measured in nT) is derived from the deviations in the horizontal component of a network of near-equatorial geomagnetic





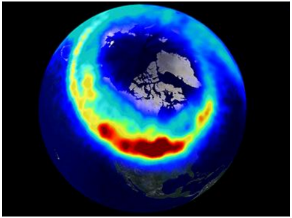
- The auroral electrojets activity is monitored by the high-latitude geomagnetic indices known as AE, AL and AU indices
- The auroral indices (measured in nT) are derived from the deviations in the horizontal component of a network ($>$10) of high-latitude geomagnetic observatories in the northern hemisphere
- They provide an estimation of the energy deposition in the auroral ionospheric regions [Ahn et al.,1983]
- AE: represents the overall activity of the auroral electrojets (AU-AL)
- AL: quantifies the current intensity variations of the westward auroral electrojet, which is mainly related to the tail activity
- AU: monitors the eastward electrojet, mainly related to the electric convection



- The magnetosphere is not an isolated system but it is continuously coupled with the solar wind
- Solar wind energy is transferred to the magnetosphere - ionosphere system and an indicator of the solar wind energy input is the Akasofu epsilon parameter

$$\epsilon = \frac{4\pi}{\mu_0} l_0^2 v B^2 sin^4(\theta_c/2) \quad [GW] \tag{1}$$

  - μ_0 is the permeability of free space
  - $l_0 = 7R_E$ is the stand-off distance of the nose of the magnetosphere
  - v is the solar wind speed
  - B is the magnitude of the solar wind magnetic field
  - θ_c is the clock angle between B_y and B_z
- $\epsilon > 10^2$ GW is likely to cause a substorm, during big storms $\epsilon > 10^4$ GW



A MULTISCALE APPROACH: THE EMPIRICAL MODE DECOMPOSITION (EMD)

- A posteriori decomposition method useful for non-linear and non-stationary datasets [Huang et al., 1998]

$$X(t) = \sum_{i=1}^{N} C_i(t) + r(t)$$

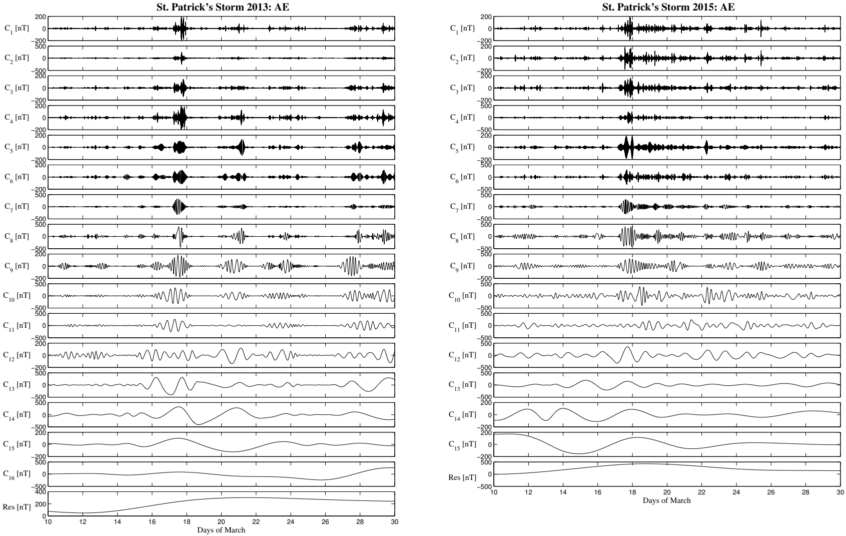
- $C_i(t)$ is called Intrinsic Mode Function (IMF) and $r(t)$ is the residue of the decomposition
- An IMF is defined as a function that:
  1. has symmetric upper and lower envelopes
  2. the number of zero crossings and the number of extrema differing at most by one.
- An IMF can be written as $C_i(t) = A_i(t)\cos[\phi_i(t)]$ where
  - $A_i(t)$ is the instantaneous amplitude
  - $\phi_i(t)$ is the instantaneous phase through which an instantaneous frequency can be derived ($\omega_i(t) = d\phi_i(t)/dt$) as well as a characteristic timescale $\tau_i = 2\pi / < \omega_i(t) >_T$

MAIN ADVANTAGES

- No "a priori" assumptions on the basis functional form
- Finite and small number of empirical modes
- Non-stationary method with time-dependent frequencies

THE MAGNETOSPHERIC DYNAMICS ...

- manifests a clear multiscale nature (Alberti et al., 2017)



THE DELAYED MUTUAL INFORMATION (DMI)

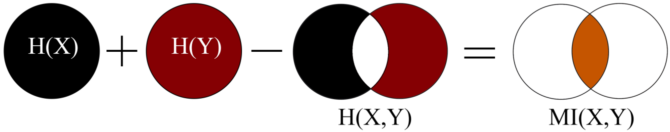
- How we can quantify the information shared between solar wind inputs and magnetospheric outputs?

Considering a time delay Δ, it is possible to introduce a quantity capable of quantifying the information shared by two sequences X(t) and Y(t) as

$$MI(X, Y|\Delta) = H(X) + H(Y) - H(X, Y|\Delta) \tag{2}$$

where

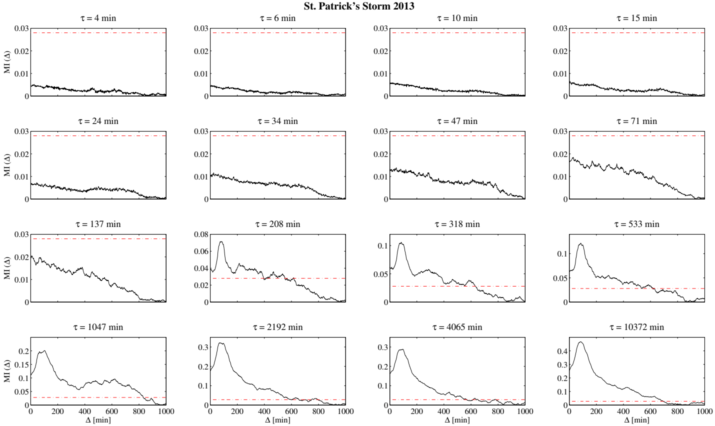
- $H(X) = -\sum_{x \in X} P(x) \log P(x)$ ($H(Y) = -\sum_{y \in Y} P(y) \log P(y)$) is the Shannon entropy
- $H(X, Y) = -\sum_{x \in X} \sum_{y \in Y} P(x, y) \log P(x, y)$ is the joint Shannon entropy



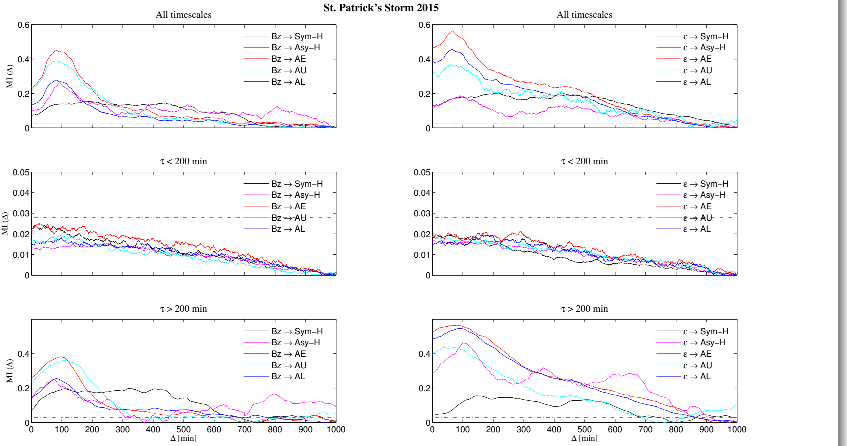
THE MAGNETOSPHERIC DYNAMICS ...

- manifests a clear separation of timescales between the internal processes and the direct driven ones, being the characteristic separation timescale of the order of 100 - 200 min (Kamide and Kokubun, 1996; Alberti et al., 2017)

SCALE-TO-SCALE DMI: $\epsilon \rightarrow$ AE



- this timescale separation is related to loading-unloading typical timescales (Consolini and De Michelis, 2005) and also with typical timescales involved in the nonlinear response of the Earth's magnetosphere (Tsurutani et al., 1990)
- internal processes can be considered more reasonably as only triggered by external solar wind changes (Alberti et al., 2017)

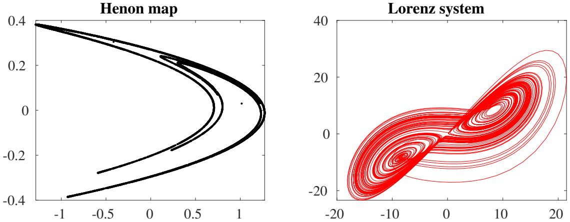


Fractal dimensions

- fractal dimensions quantify complexity (i.e., changing detail with changing scale)
- a fractal dimension does not have to be an integer

Strange attractors

- an **attractor** is a set of numerical values toward which a system tends to evolve, for a wide variety of initial conditions (a point, a curve, a manifold)
- mathematically is a subset $\mathbb{A}$ of the phase space characterized by the properties:
  1. $\mathbb{A}$ is *invariant*: if $a \in \mathbb{A}$ then $f(t, a) \in \mathbb{A}$, $\forall t > 0$;
  2. $\mathbb{A}$ *attracts an open set of initial conditions*: there exists the basin of attraction $\mathbf{B}(\mathbb{A})$;
  3. $\mathbb{A}$ is *minimal*: there is no proper subset of $\mathbb{A}$ having the first two properties.
- if $\mathbb{A}$ is a complicated set with a fractal structure, also exhibiting a sensitive dependence of initial conditions, then it is known as a strange attractor



GENERALIZED DIMENSIONS D_q ($q > 0$) *(Hentschel & Procaccia, 1983)*

- 1918: Hausdorff proposed “to measure” fractals by using the **Hausdorff dimension≡fractal dimension**
- 1980-1981: several authors (Grassberger, Procaccia, Takens, etc.) proposed *only* three dimensions:
  1. the box-counting dimension $\mathbf{D_0}$
  2. the information dimension $\mathbf{D_1}$
  3. the correlation dimension $\mathbf{D_2}$
- 1983: Hentschel and Procaccia proved that fractals and strange attractors are characterized by an **infinite number of generalized dimensions**

Characterization of Strange Attractors

Peter Grassberger[(a)] and Itamar Procaccia
Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 7 September 1982)

A new measure of strange attractors is introduced which offers a practical algorithm to determine their character from the time series of a single observable. The relation of this new measure to fractal dimension and information-theoretic entropy is discussed.

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MEASURING THE STRANGENESS OF STRANGE ATTRACTORS

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Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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Revised 26 May 1983

We study the correlation exponent ν introduced recently as a characteristic measure of strange attractors which allows one to distinguish between deterministic chaos and random noise. The exponent ν is closely related to the fractal dimension and the information dimension, but its computation is considerably easier. Its usefulness in characterizing experimental data which stem from very high dimensional systems is stressed. Algorithms for extracting ν from the time series of a single variable are proposed. The relations between the various measures of strange attractors and between them and the Lyapunov exponents

THE INFINITE NUMBER OF GENERALIZED DIMENSIONS OF FRACTALS AND STRANGE ATTRACTORS

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Revised 30 March 1983

We show that fractals in general and strange attractors in particular are characterized by an infinite number of generalized dimensions D_q, $q > 0$. To this aim we develop a rescaling transformation group which yields analytic expressions for all the quantities D_q. We prove that $\lim_{q\to 0} D_q$ = fractal dimension (D), $\lim_{q\to 1} D_q$ = information dimension (σ) and $D_{q=2}$ = correlation exponent (ν). D_q with other integer q's correspond to exponents associated with ternary, quaternary and higher correlation functions. We prove that generally $D_q > D_{q'}$ for any $q' > q$. For homogeneous fractals $D_q = D_{q'}$. A particularly interesting dimension is $D_{q=\infty}$. For two examples (Feigenbaum attractor, generalized baker's transformation) we calculate the generalized dimensions and find that D_∞ is a non-trivial number. All the other generalized dimensions are bounded between the fractal dimension and D_∞.

KOLMOGOROV ENTROPY

- assuming to have a $d-$dimensional space partioned into cubes of size ℓ^d
- let be Δt the sampling of a long time series $\{\mathbf{X}_i\}_{i=1}^N$

$$K_2 = -\lim_{\Delta t \to 0} \lim_{\ell \to 0} \lim_{N \to \infty} \frac{1}{N \Delta t} \sum_{i_1, i_2, \ldots, i_N} p(i_1, i_2, \ldots, i_N) \log p(i_1, i_2, \ldots, i_N)$$

- p is the joint probability that $\mathbf{X}(t = \Delta t)$ is in the box i_1, $\mathbf{X}(t = 2\Delta t)$ is in the box i_2, $\ldots$, $\mathbf{X}(t = N\Delta t)$ is in the box i_N
- K_2 is a measure of the rate of loss of information, since K_2^{-1} is the timescale over which the behavior of the system can be accurately predicted, as well as it is a measure of sensitivity of the system to changes in initial conditions
- if K_2 is finite, the system is chaotic, while if $K_2 \to \infty$, the system is nondeterministic

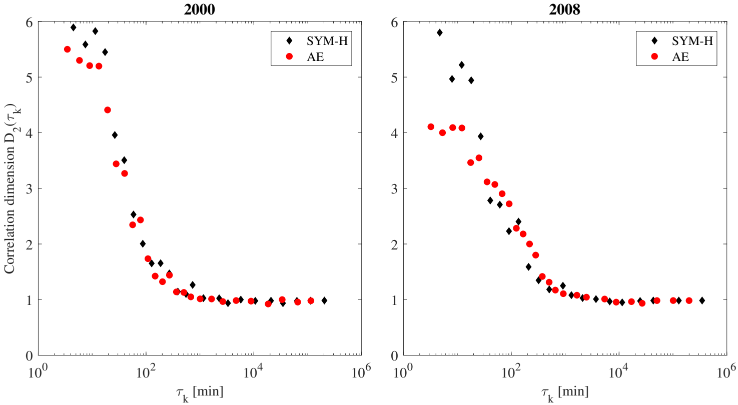
K_2 VS. D_2

- let m be the embedding dimension and Δ the time delay to construct a $m-$component state vector from the time series $\{\mathbf{X}_i\}_{i=1}^N$

$$K_2 = \frac{1}{\Delta t} \lim_{\ell \to 0} \log \frac{C(\ell, m)}{C(\ell, m+1)} \quad \text{being} \quad C(\ell) = \frac{1}{N^2} \sum_{i \neq j} \Theta(\ell - |\mathbf{X}_i - \mathbf{X}_j|)$$

FROM A DYNAMICAL SYSTEM POINT OF VIEW ...

- the overall magnetospheric dynamics has been described in terms of a low-dimensional chaotic system (Vassiliadis et al., 1990)
- this view does not take into account the dynamical changes on different timescales
  - ✓ Kolmogorov entropy and correlation dimension D_2 are scale-dependent
  - ✗ forecast horizon for fast dynamics is ∼2 min ⇒ we need to have high-dimensional models ($D_2 \sim 4-5$, Consolini et al., 2018)



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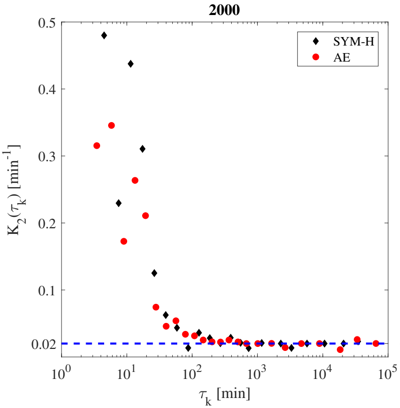

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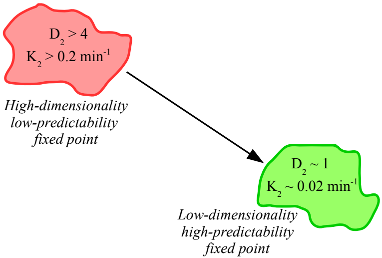
- fast and slow dynamics are governed by different fixed points, characterized by a different number of degrees of freedom
- the emerging scenario is that in presence of a sort of topological continuous phase transition for the fluctuations at different timescales (Chang et al., 1992, 2003; Consolini et al., 2018)



A STOCHASTIC DESCRIPTION ...

- The magnetosphere can be described in terms of a simple nonlinear system with many dynamical states by means of a 1-D Langevin model

$$dx = -\frac{\partial U(x)}{\partial x} dt + \sigma dW \qquad (3)$$

where x is the state variable, $U(x)$ is the state function, σ is the noise level and W is a Wiener process.

- The associated Fokker-Planck equation reads

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[U'(x)\rho(x,t)\right] + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial x^2}\rho(x,t) \qquad (4)$$

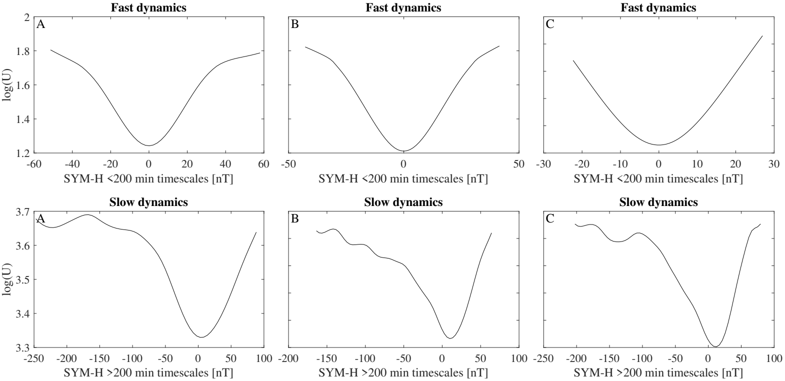
- its stationary solution is

$$\rho(x) \sim exp\left[-\frac{2U(x)}{\sigma^2}\right] \rightarrow U(x) = -\frac{\sigma^2}{2}\ln \rho(x) = \sum_{i=0}^{L} a_i x^i \qquad (5)$$

- L is related to the number of states which is $L/2$

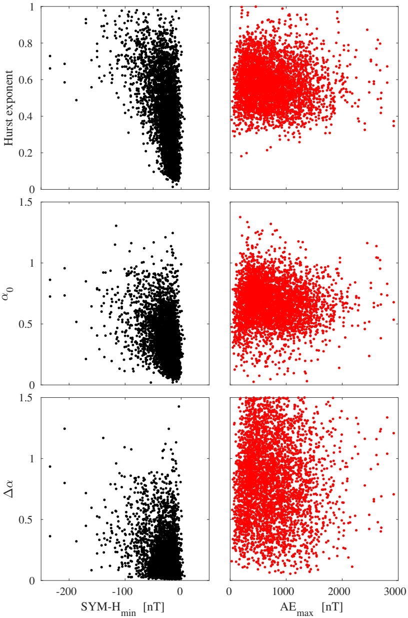
A STOCHASTIC DESCRIPTION ...

- ✓ the slow dynamics related to interplanetary changes is characterized by a multi-state dynamics (Alberti et al., 2018)
- ✓ the fast dynamics is characterized by a quasi-invariant single-state dynamics



- ✓ meta-stable states occur during disturbed periods
- ✓ a single stable state is found during quiet periods at all timescales

A FRACTAL DESCRIPTION ...



- use time series over the 24th solar cycle
- use a moving time window of length 2 days ($N_p = 2880$, $q = 3$) to evaluate
  - the Hurst exponent $\mathcal{H}$
  - the Hölder exponent α_0 corresponding to $\frac{df}{d\alpha}|_{\alpha=\alpha_0} = 0$
  - the singularity width $\Delta\alpha = \alpha_{max} - \alpha_{min}$

- a different pattern is clearly observed between SYM-H and AE
- larger $\mathcal{H}$ and $\Delta\alpha$ are found for AE
- singularities increase with geomagnetic activity for SYM-H
- $\mathcal{H}$ also increases with geomagnetic activity for SYM-H
- complexity changes during a storm?
- what about substorm-storm relationships?

CONCLUSIONS

- a low-dimensional system cannot explain fast dynamics occuring on timescales <200 min
- a high predictability has been found for the slow dynamics (>200 min) which is directly driven from interplanetary changes (Alberti et al., 2017, 2018)
- new light on the framework of Space Weather forecasting:
  - ✗ all "deterministic" models (as based on neural networks) fail in reproducing the fast dynamics which is the most critical for Space Weather purposes
  - ✓ it is quite reasonable to get a good forecast of that part of the magnetospheric dynamics associated with the enhancement convection processes
  - ✓ the fast dynamics associated with the unloading mechanisms taking place in the CPS and neutral sheet tail regions requires a deeper knowledge of the magnetospheric tail conditions

TIPS

- more work will need to be done to determine some proxies for the tail dynamical state with a time resolution of seconds
- the individuation and the construction of a proxy for the internal fast dynamics should be considered as a must
- the fast dynamics is responsible for a lot of phenomena such as the generation of the large ground-induced currents

THANKS FOR THE ATTENTION . . .

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