



# Characterizing magnetic reconnection using Gaussian Mixture Models on electron particle velocity distributions

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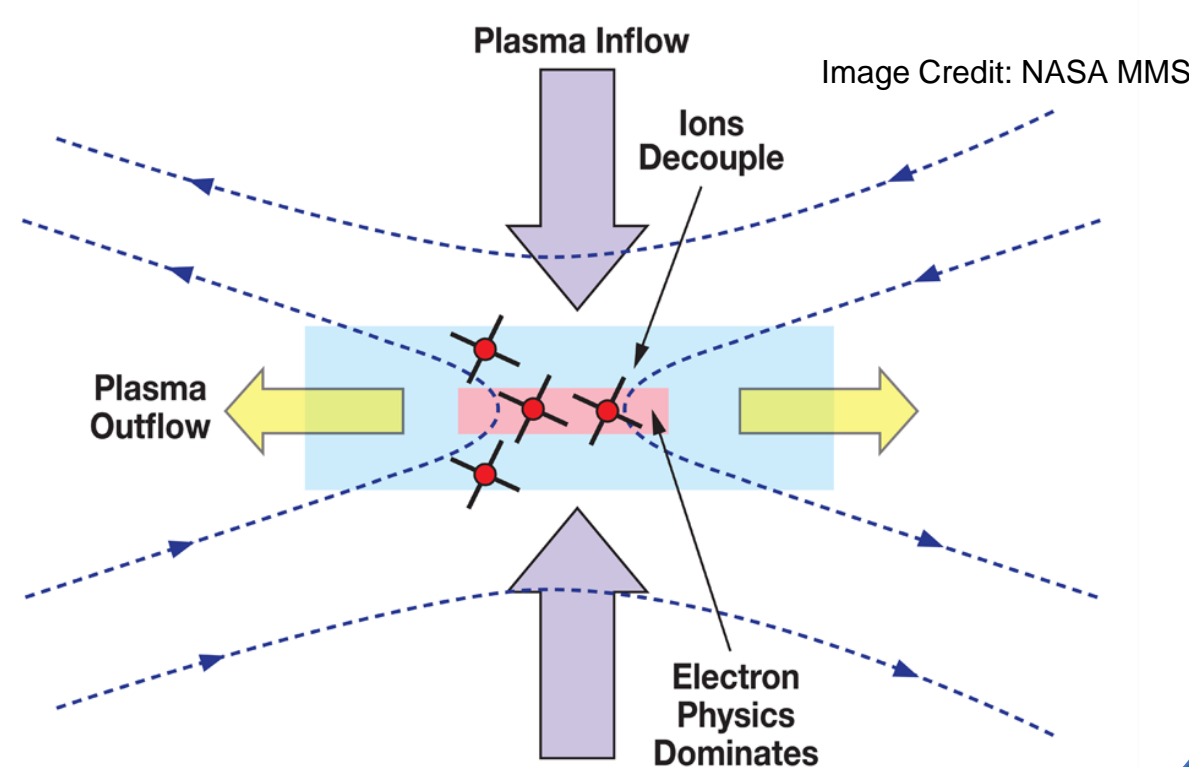
We present a method based on unsupervised machine learning to automatically identify and characterize regions of interest using particle velocity distributions as a signature pattern. A density estimation technique is applied to particle distributions provided by PIC simulations to study magnetic reconnection regions. The key components of the method involve: i) a Gaussian Mixture Model determining the presence of a given number of subpopulations within an overall population, and ii) a model selection technique with Bayesian Information Criterion to estimate the appropriate number of subpopulations. Thus, this method identifies automatically the presence of complex distributions, such as beams or other non-Maxwellian features, and can be used as a detection algorithm able to identify reconnection.

## Magnetic reconnection

Reconnection plays a crucial role in collisionless plasmas. By breaking down the frozen-in magnetic fields, the magnetic energy is converted into kinetic, energy, and particle acceleration energy. Reconnection occurs at various locations such as laboratory plasma, solar atmosphere, or the magnetotail.

This process represents one of the most important sources of particle acceleration in space.

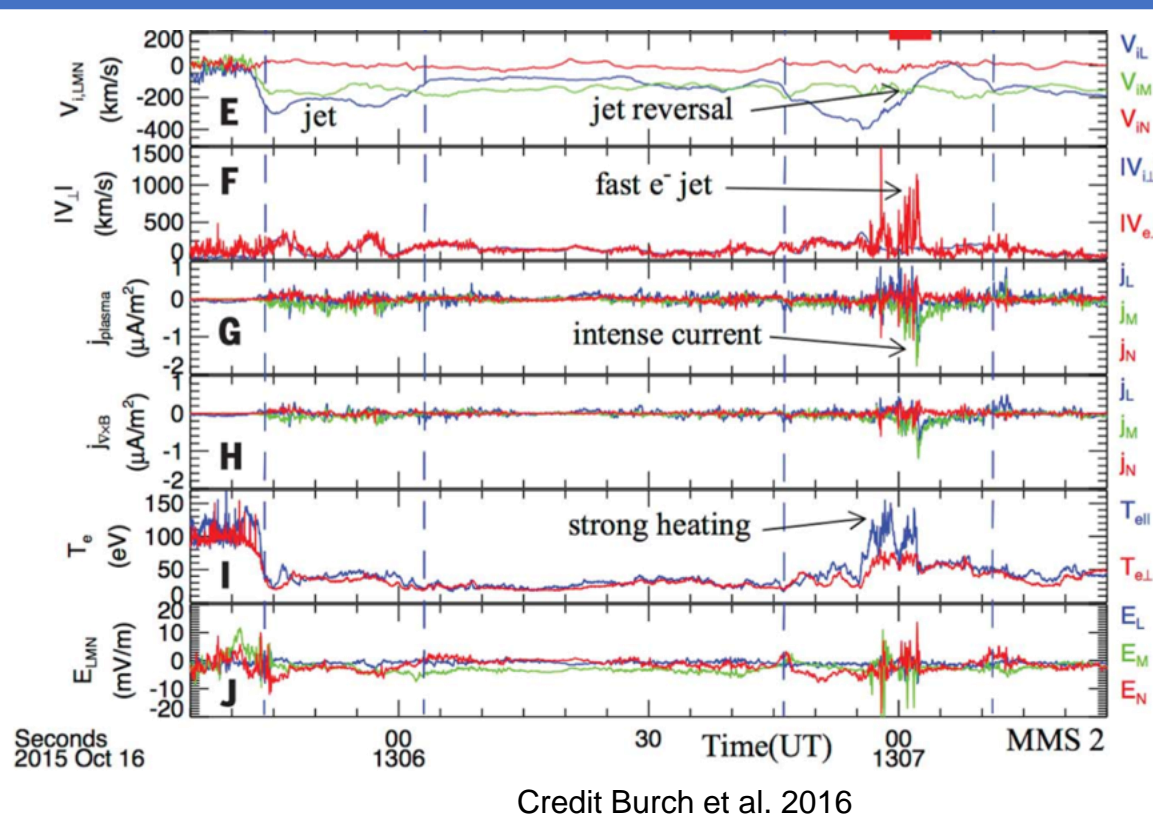
Magnetic reconnection is studied with in-situ measurements (MMS mission [1]) and simulations [2].



## Challenging detection from data

**Simulations:** thousands of particles per cell with a total data size nowadays reaching trillions of particles defined as 6D data.

**Observations:** distributions are available at specific time ranges defined manually by an operator or basic rules using to fields data.



Metrics have been developed from statistical moments of the distributions. One can cite the measure of gyrotropy [3] and the agyrotropy.

**Our approach: using directly the full distribution**

## Gaussian Mixture Model

The probability density function is approximated by a sum of Gaussians  $\mathcal{N}$ :

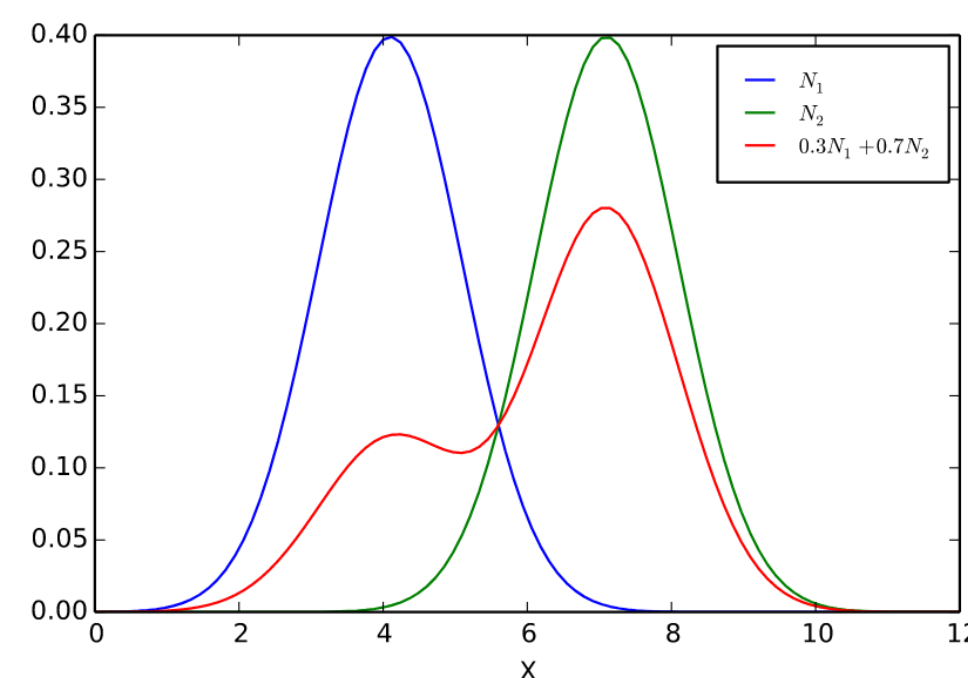
$$p(\mathbf{x}|\Phi) = \sum_{k=1}^K w_k \mathcal{N}(\mathbf{x}|\theta_k)$$

$\theta_k$  k-th mean and covariance  
 $w_k$  weight of the k-th Gaussian  
 $K$  the number of Gaussians

$$\Phi = [w_1, \dots, w_K, \theta_1, \dots, \theta_K]$$

The vector  $\Phi$  is computed as follow:

- 1) Discretize density by a list of particles
- 2) The likelihood function  $L$  is maximized
- 3) Expectation-Maximization algorithm
- 4) No closed form, only local maximums



**Model selection problem:** How to determine K in advance?

Bayesian Information Criterion (BIC) is maximized among a set of models

$$BIC = \underbrace{\ln(n)k}_{\text{complexity}} - \underbrace{2 \ln(L)}_{\text{goodness of fit}}$$

$k$  number of free parameters  
 $n$  number of particles

The value K does not necessary correspond to the number of subpopulations:

- It may be beams or electron subpopulation
- It can also reflects complex distributions (flat-top, Kappa, crescent shape, etc.)

## Diagnostic tool

Thermal energy as the second moment may be inappropriate if applied on several subpopulations:

$$E_{thermal} = \frac{1}{N_p} \sum_{i=1}^3 \left[ \sum_p (\mathbf{V}_p - \langle \mathbf{V}_p \rangle)^2 \right]_i$$

The second statistic moment on the mixture has additional terms:

$$(\sigma^2)^{(K)} = \sum_{i=1}^3 \left[ \sum_{k=1}^K w_k^2 (\sigma_k)^2 + \sum_{k=1}^K w_k (\mu_k)^2 - \left( \sum_{k=1}^K w_k (\mu_k) \right)^2 \right]_i$$

$E_{thermal}^{(K)}$   
 $E_{dev}^{(K)}$

$E_{thermal}^{(K)}$  relates to the thermal energy of the mixture. The ratio with the energy shows the reduction in thermal speed (heating vs acceleration)

$E_{dev}^{(K)}$  is the weighted variance of the mixture means measuring. The ratio with the energy highlights the complexity of the distribution.

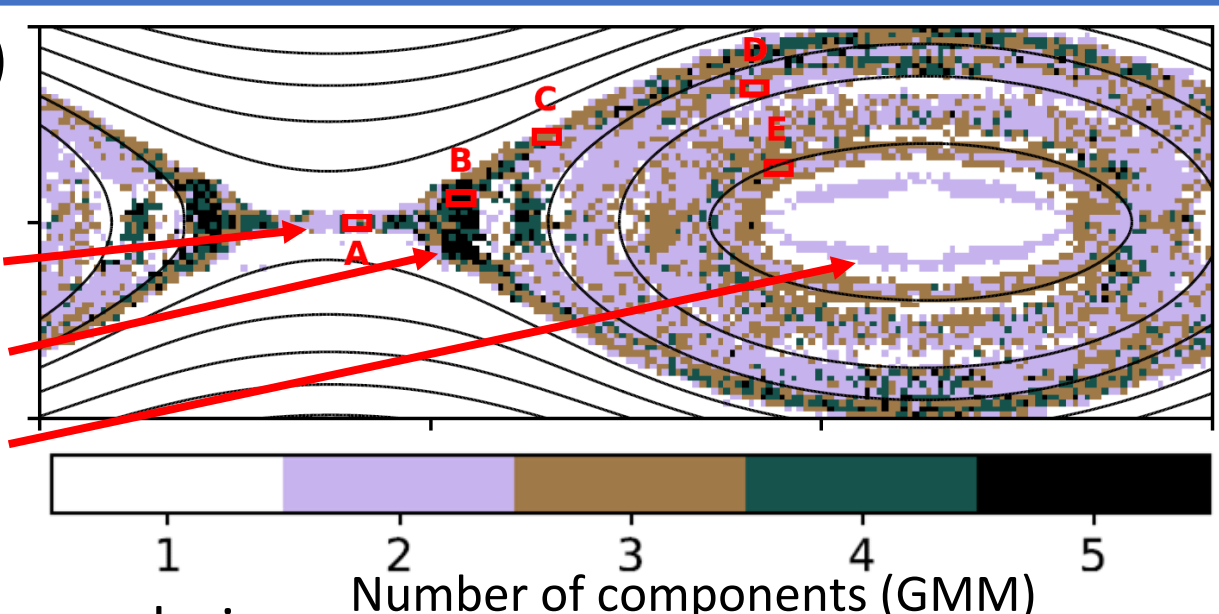
$$E_{drop} = \frac{E_{thermal}^{(K)}}{E_{thermal}}$$

$$E_{dev} = \frac{E_{dev}^{(K)}}{E_{thermal}}$$

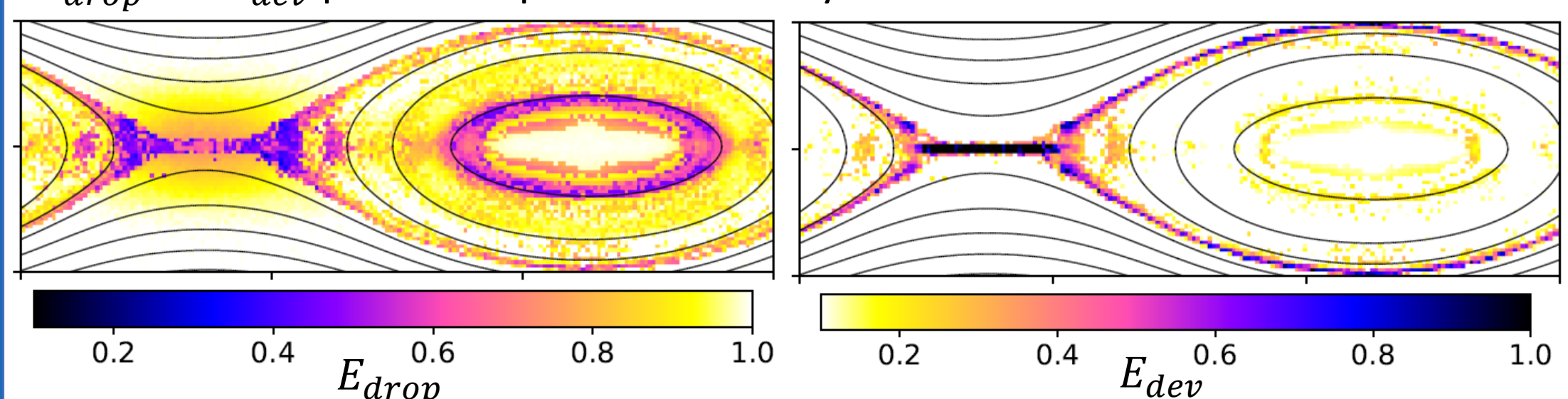
## Results

2.5D Particle-In-Cell (PIC) simulation, double Harris sheet case

Electron diffusion region (EDR)  
Outflow  
O-point



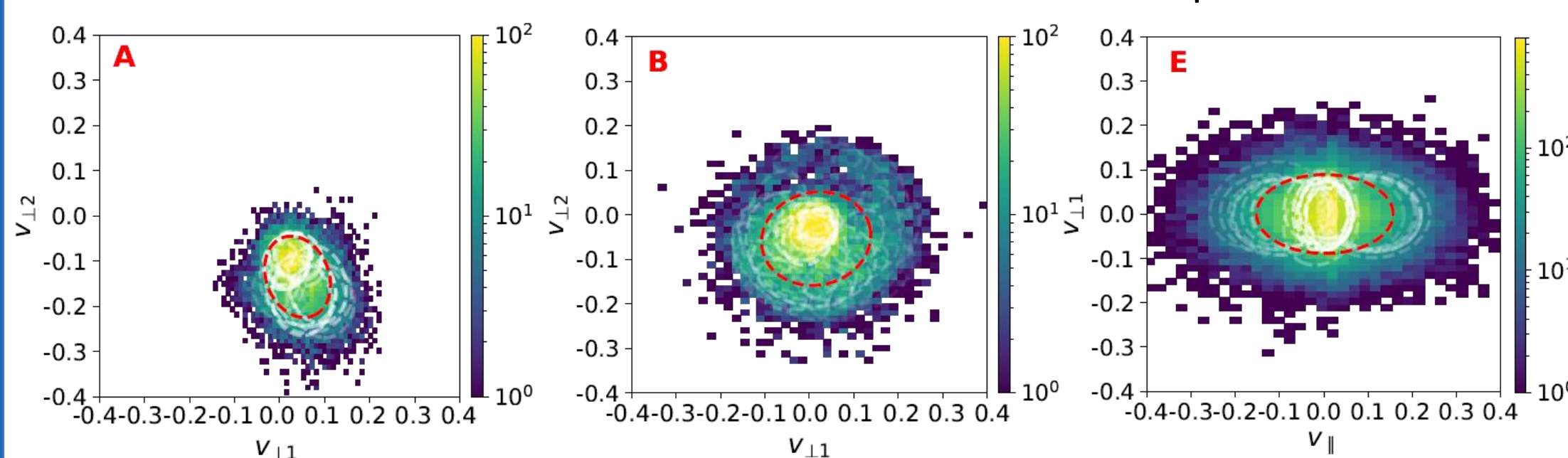
$E_{drop}$  and  $E_{dev}$  provide a quantitative analysis



Several regions are identified. The inflow is characterized by a large region of high value. The EDR and the outflow show low values, meaning the mixture extend is reduced compared to the second moment. A ring is also observed near the O-point.

A very narrow  $E_{dev}$  peak is observed around the EDR, suggesting complex distributions are present.

Typical topological boundaries are highlighted by  $E_{dev}$ , in particular at the outflow, near the separatrix and at the limit far from the O-point.



The above distributions refer to the labels in the first figure of the section. They illustrate various behaviors, such as a Gaussian core associated with a surrounding halo (A and B) or an anisotropic heavy-tail distribution (E).

<https://arxiv.org/abs/1910.10012>

[1] Burch et al., "Magnetospheric multiscale overview and science objectives", Space Science Reviews, 2016

[2] Goldman et al. 2016, "What can we learn about magnetotail reconnection from 2D PIC Harris-sheet simulations?" Space Science Reviews, 199, 651

[3] Swisdak, "Quantifying gyrotropy in magnetic reconnection", Geophysical Research Letters